Decentralizing the Optimal Multi-cell Beamforming via Large System Analysis

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Abstract—**A multi-cell minimum power beamforming problem is considered. It is known that the inter-cell interference (ICI) terms couple the base stations (BS) and inter-cell coordination is required for global optimal solution. The cooperation can be realized by exchange of instantaneous channel state information (CSI) or terms related to the ICI values via a backhaul link. However, the limited backhaul capacity and delay constraints put a limit on achievable performance when the number of antennas and users grow large or when dealing with a fast fading scenario. In this work, we demonstrate that the ICI terms coupling the coordinating BSs can be approximated using the random matrix theory (RMT) tools when the problem dimensions grow large, and that the approximated ICI values depend only on channels statistics, i.e., spatial load and user specific path loss values. This leads to a significant reduction in the information exchange rate among BSs. Furthermore, processing is simplified because with a fixed approximated ICI values the beamforming vectors can be obtained locally at each BS. The proposed solution guarantees the feasibility of the target signal-to-interference-plus-noise ratios (SINR) without any major loss of performance as compared to the optimal centralized design.**

I. INTRODUCTION

Coordinated multi-point transmission (CoMP) allows cooperation and coordination between nodes for delivering services to users which results in greatly improved optimization objective values as compared to the non-coordinated transmission. In order to realize the gains in practice, the coordinating base stations must share some information. Coordinated multi-cell minimum power beamforming approach considered in this paper satisfies a given signal-to-interference-plus-noise ratio (SINR) for all users while minimizing the total transmitted power. Sharing instantaneous CSI between nodes under a delay constraint and limited backhaul capacity becomes a principal problem when dimensions of the problem (number of antennas N and number of users K) grow large or when dealing with a fast fading scenario. Authors in [1] have extended the work in [2] for a large dimension system where random matrix theory (RMT) is utilized to give approximate beamformers at each BS, which rely only on the local CSI and average channel statistics (spatial load, pathloss information) from the other BS channels. However, the target SINR feasibility cannot be guaranteed as the error in approximations is translated into variation in the resulting SINR values.

In [3]–[5], an alternative decentralized framework is proposed for the coordinated multi-cell minimum power beamformer design problem. The optimal minimum power beamformers can be obtained locally at each base station (BS) relying on limited backhaul information exchange between BSs. The proposed method is able to guarantee feasible solutions even if the interference information is outdated or incomplete.

In this work, the focus is on large-scale multiple antenna wireless systems with a large number of low-power antennas co-located at the BS site, often called as *massive MIMO* [6], [7]. One important benefit of such a setting is that some of the analysis can be carried out using tools from random matrix theory (RMT) [7]. It was shown in [6] that with very large imbalance $N >> K$, the processing can be simplified in a way that even matched filter (MF) and zero-forcing (ZF) can be used in an ideal i.i.d. channel for near optimal detection and beamforming [6], [7]. However, in practical multi-cell environments with non-ideal, correlated channels, the use of more complicated precoder design algorithms is justified as the performance gains compared to simple MF or ZF based schemes are still significant. Following the same logic as in [3], inter-cell interference (ICI) is considered as the principal coupling parameter among BSs. We use a large dimension approximation for ICI term based on random matrix theory which leads to a distributed beamforming algorithm. This algorithm gives the feasible beamforming vectors at each BS based on the local CSI only, while the coupling ICI terms are based on channel statistics (spatial load, propagation loss values) from the other BS channels. The proposed algorithm benefits from the large dimension simplifications which results in almost optimal transmit powers along with significant reduction in the backhaul exchange rate.

II. SYSTEM MODEL

A cellular system is considered which consists of N_{B} BSs, each BS has N_{a} transmit antennas. Each user has a single receive antenna. Users allocated to the b^{th} base station are in set $\mathcal{U}_b$. Each user is served by a single base station and the BS that serves user k is denote by b_k. Sets of all users and all BSs are presented by $\mathcal{U}$ and $\mathcal{B}$ respectively. The received signal for user k can be presented as,

$$\mathbf{y}[k] = \mathbf{h}_{b_k,k}\mathbf{x}_{b,k} + \mathbf{h}_{b_k,k} \sum_{l\neq k \in \mathcal{U}_{b_k}} \mathbf{x}_{b,l} + \sum_{b\neq b_k} \mathbf{h}_{b,k} \sum_{l\in\mathcal{U}_b} \mathbf{x}_{b,l} + \mathbf{n}_k \quad (1)$$

where $\mathbf{n}_k \sim \mathcal{CN}(0, N_0)$ is the noise and $\mathbf{h}_{b,k} \in \mathbb{C}^{1\times N_{\mathrm{a}}}$ represents the channel from the b^{th} BS to k^{th} user. The path-losses $a^2_{b,k}$ are included in the channel vectors, i.e, $\mathbf{h}_{b,k} \sim \mathcal{CN}(0, a^2_{b,k}\mathbf{I}_{\mathrm{N_a}})$. $\mathbf{x}_{b,k} = \mathbf{w}_{b,k} d_k$ is the transmitted vector from the b^{th} BS to k^{th} user, in which d_k is the normalized complex data symbol ($E[|d_k|^2] = 1$) and $\mathbf{w}_{b,k} \in \mathbb{C}^{N_{\mathrm{a}}}$ is the downlink beamforming vector from the b^{th} BS to k^{th} user.

III. PROBLEM FORMULATION

The optimization problem for achieving the optimal downlink beamformers as proposed by [3] can be presented as

$$\begin{aligned} \underset{\mathbf{w}_{b,k},\epsilon_{b,k}}{\text{minimize}} \quad & \sum_{b\in\mathcal{B}} \sum_{k\in\mathcal{U}_b} \|\mathbf{w}_{b,k}\|^2 \\ \text{subject to} \quad & \Gamma_k \geq \gamma_k \ \forall k \in \mathcal{U}_b, \forall b, \\ & \sum_{l\in\mathcal{U}_b} |\mathbf{h}_{b,k}\mathbf{w}_{b,l}|^2 \leq \epsilon^2_{b,k}, \forall k \notin \mathcal{U}_b, \forall b, \end{aligned} \quad (2)$$

where the intercell interference from b^{th} base station to user k is denoted by $\epsilon^2_{b,k}$, and where

$$\Gamma_k = \frac{|\mathbf{h}_{b_k,k}\mathbf{w}_{b_k,k}|^2}{N_0 + \sum_{l\in\mathcal{U}_{b_k}\setminus k} |\mathbf{h}_{b_k,k}\mathbf{w}_{b_k,l}|^2 + \sum_{b\neq b_k} \epsilon^2_{b,k}} \quad (3)$$

The centralized problem in (2) is decoupled among BSs as soon as the ICI terms $\epsilon_{b,k}$ are set to fixed values. Decentralized solutions have been proposed based on dual [3] or primal decomposition [4], [5], or alternating direction method of multipliers (ADMM) [8].

Another approach for solving the optimization problem defined by (2) is based on uplink-downlink duality. Authors in [2] have shown that the problem dual to (2) which gives the optimal uplink power allocation and detection vectors is defined as follows

$$\begin{aligned} \underset{\hat{\mathbf{w}},\lambda}{\text{minimize}} \quad & \sum_{b\in\mathcal{B}}\sum_{k\in\mathcal{U}_b} \lambda_k N_0 \\ \text{subject to} \quad & \frac{\lambda_k|\hat{\mathbf{w}}^{\mathrm{H}}_{b_k,k}\mathbf{h}^{\mathrm{H}}_{b_k,k}|^2}{\sum_{l\neq k}\lambda_l|\hat{\mathbf{w}}^{\mathrm{H}}_{b_l,l}\mathbf{h}^{\mathrm{H}}_{b_l,k}|^2+\|\hat{\mathbf{w}}_{b_k,k}\|^2}\geq\gamma_k \ \forall k\in\mathcal{U} \end{aligned} \tag{4}$$

The dual uplink power of the k^{th} user is denoted by λ_k that its optimal value can be calculated by a fixed point iteration [2]

$$\lambda_k = \frac{1}{(1+\frac{1}{\gamma_k})\mathbf{h}_{b_k,k}(\mathbf{\Sigma}_{b_k}+\mathbf{I})^{-1}\mathbf{h}^{\mathrm{H}}_{b_k,k}} \tag{5}$$

where $\mathbf{\Sigma}_b = \sum_{l\in\mathcal{U}}\lambda_l\mathbf{h}^{\mathrm{H}}_{b,l}\mathbf{h}_{b,l}$. The dual uplink detection vector $\hat{\mathbf{w}}_{b,k}$ is given by the minimum mean square error receiver at the optimal point [2], i.e, $\hat{\mathbf{w}}_{b,k} = (\mathbf{\Sigma}_b+\mathbf{I})^{-1}\mathbf{h}^{\mathrm{H}}_{b,k}$. A link between the downlink and uplink beamformers is provided by $\mathbf{w}_{b_k,k} = \sqrt{\delta_k}\hat{\mathbf{w}}_{b_k,k}$ where $\mathbf{w}_{b,k}$ is the downlink beamformer for the k^{th} user and δ_k can be found by the following matrix inversion [2]

$$G_{i,j} = \begin{cases} \frac{1}{\gamma_i}|\hat{\mathbf{w}}^{\mathrm{H}}_{b_i,i}\mathbf{h}^{\mathrm{H}}_{b_i,i}|^2 & i=j \\ -|\hat{\mathbf{w}}^{\mathrm{H}}_{b_j,j}\mathbf{h}^{\mathrm{H}}_{b_j,i}|^2 & i\neq j. \end{cases} \tag{6}$$

Finally, $\boldsymbol{\delta} = \mathbf{G}^{-1}\mathbf{1}_{Nu}$ where $\boldsymbol{\delta}$ is a vector that contains all δ_k values and $\mathbf{1}_{Nu}$ is a $Nu\times 1$ vector with all elements equal to one. Note that the final step requires a global knowledge about the CSI which makes its distributed implementation difficult, especially when dealing with a large number of users and antennas.

IV. DECENTRALIZED APPROACH FOR LARGE DIMENSION SYSTEM

In this section we introduce our decentralized algorithm for a system with large dimensions based on the approximated ICI thresholds. Authors in [1] have considered a system with large dimensions, i.e., large number of users and antennas. This assumption allows deriving a large dimension approximation for (5) and (6) that results in a decentralized beamforming approach which relies on local CSI and the average statistics of the other channels. According to [1], an approximation for the optimal uplink power defined by (5) can be formulated as

$$\lambda_k = ((1+\frac{1}{\gamma_k})(\frac{a^2_{b_k,k}m_{\mathbf{\Sigma}_{b_k}}(-1)}{1+a^2_{b_k,k}\lambda_k m_{\mathbf{\Sigma}_{b_k}}(-1)}))^{-1} \tag{7}$$

where, $a^2_{b_k,k}$ is the pathloss from the k^{th} user to its serving base station. $m_{\mathbf{\Sigma}_{b_k}}(-1)$ is the Stieltjes transform of the Gram matrix $\mathbf{\Sigma}_{b_k}$ at point $z=-1$ [9].

Similarly, the matrix inversion for δ_k values can be approximated by [1]

$$G_{i,j} = \begin{cases} \frac{1}{\gamma_i}(\frac{a^2_{b_i,i}m_{\mathbf{\Sigma}_{b_i}}(-1)}{\eta_{b_i,i}})^2 & i=j \\ \frac{-1}{N_{\mathrm{a}}}\frac{a^2_{b_j,i}a^2_{b_j,j}m'_{\mathbf{\Sigma}_{b_j}}(-1)}{\eta^2_{b_j,i}\eta^2_{b_j,j}} & i\neq j \end{cases} \tag{8}$$

where $\eta_{b_j,i} = 1+a^2_{b_j,i}\lambda_i m_{\mathbf{\Sigma}_{b_j}}(-1)$ and where $m'_{\mathbf{\Sigma}_b}(-1)$ is the differential of $m_{\mathbf{\Sigma}_b}(z)$ with respect to z at point $z=-1$.

Under the large dimension assumption, the beamforming vectors can be found using locally acquired channel knowledge and approximated dual uplink powers (7) and cross-coupling matrix (8) instead of (5) and (6), resulting in a distributed beamforming algorithm. However, the problem with the approximated method is that it cannot guarantee the target SINRs for finite number of antennas as the error in approximations is translated into variations in the resulted SINRs which can be less or more than the target SINRs.

A. *Approximation of intercell interference terms*

The method proposed here relies on approximately optimal ICI values instead of approximated uplink powers as in [1]. The approximate ICI values remain valid for a given set of users until a change occurs in the statistics of the channel, i.e., when a user changes its location. This leads potentially to a significant reduction of the required backhaul signaling depends only on the large scale parameters, i.e. pathloss between each BS and active node.

The large dimension approximation for ICI terms can be achieved by using (7) and (8). From (2) and (3), it is clear that the intercell interference from all the base stations towards user k is

$$\sum_{b\neq b_k}\epsilon^2_{b,k} = \sum_{b\in\overline{\mathcal{B}_k}}\sum_{l\in\mathcal{U}_b}|\mathbf{h}_{b,k}\mathbf{w}_{b,l}|^2. \tag{9}$$

The intercell interference term in (9) can be written as follows,

$$\epsilon^2_{b,k} = \sum_{l\in\mathcal{U}_b}\sqrt{\delta_{b,l}}|\mathbf{h}_{b,k}\hat{\mathbf{w}}_{b,l}|^2 \tag{10}$$

and the approximation for the cross-terms $|\mathbf{h}_{b,i}\hat{\mathbf{w}}_{b,j}|^2$ are defined by (8),

$$|\mathbf{h}_{b,i}\hat{\mathbf{w}}_{b,j}|^2 \approx \frac{1}{N_{\mathrm{a}}}\frac{a^2_{b_j,i}a^2_{b_j,j}m'_{\mathbf{\Sigma}_{b_j}}(-1)}{\eta^2_{b_j,i}\eta^2_{b_j,j}} \quad i\neq j \tag{11}$$

Therefore, the ICI from the b^{th} BS to the k^{th} user can be written as,

$$\epsilon^2_{b,k} = \sum_{l\in\mathcal{U}_b}\sqrt{\delta_{b,l}}\frac{1}{N_{\mathrm{a}}}\frac{a^2_{b_l,k}a^2_{b_l,l}m'_{\mathbf{\Sigma}_{b_l}}(-1)}{\eta^2_{b_l,k}\eta^2_{b_l,l}} \tag{12}$$

This approximation allows derivation of approximate optimal ICI based on statistics of the user channels. Each BS needs knowledge about user specific average statistics, i.e., pathloss values from other BSs based on which each BS can locally and independently calculate the approximately optimal ICI values.

The proposed algorithm guarantees the target SINRs because the feasible solution of the optimization problem defined by (2) always satisfies the constraints and the possible error in approximations is translated into a somewhat higher transmit power at BSs compared to the optimal centralized solution.

V. NUMERICAL EXAMPLE

The algorithm developed in the previous section satisfies the target SINRs for all users; however, the error in approximations results a higher transmit power at BSs. In order to evaluate the difference between the optimal transmit power and the power resulted from the approximated algorithm, an extensive multi-cell simulation study is carried out in this section. A network with 7 cells is considered and users are scattered on the coverage area of the network, in a way that each cell contains 4 users. Exponential pathloss model is used for assigning the pathloss to each user.

Fig. 1 illustrates the transmit powers versus the number of antennas for 0dB and 10dB SINR target, respectively. The fading characteristics per antenna is i.i.d. It is clear that the gap between the approximate and optimal algorithm (denoted as SOCP) diminishes as the number of antennas increases. When the number of antennas is equal to 28 the gap is less than 0.5dB which indicates that the approximate algorithm provides a good solution for the practical scenarios with a limited number of antennas.

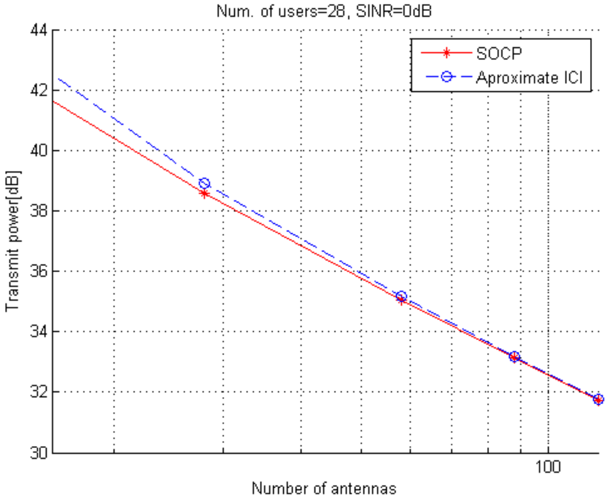

Fig. 1: Required transmit power for 0 dB SINR target.

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