Compressed Data Acquisition and Progressive Signal Recovery in Wireless Sensor Networks

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***Abstract*—We consider compressed acquisition and progressive reconstruction of spatially and temporally correlated data in wireless sensor networks (WSNs). We propose a novel, sliding window based compressed sensing (CS) method in which the sink can instantaneously reconstruct WSN samples from periodically acquired CS measurements. Moreover, the prior information attained by decoding the WSN readings multiple times allows the method to progressively refine the signal estimates. Numerical results exemplify that our proposed method results in higher CS recovery accuracy, yet with lower decoding delay and complexity, as compared to the state of the art methods.**

I. Introduction

Wireless sensor networks (WSNs) with multiple battery-powered sensors have been frequently proposed for different data gathering applications, in which the readings typically encompass spatial and temporal correlation. Especially, compressed sensing (CS) [1]–[4] has shown great potential in correlated data gathering multi-hop WSNs [5]–[8]. Namely, CS allows to accurately reconstruct a compressible signal from fewer measurements than the original signal dimension.

Aggregated delivery of spatially correlated data can be achieved, e.g., by linearly combining sensor measurements along multi-hop routing [5], [6], or by acquiring only a fraction of sensor readings [7]. Correspondingly, temporal correlation may allow each sensor to transmit linear projections of a block of its samples [9]. For spatio-temporal correlation, joint sparsity models [10] were proposed to model particular joint correlation structures, and [11] introduced Kronecker compressed sensing for acquiring multi-dimensional signals with general correlation patterns. Applying CS for blocks of data inevitably induces decoding delay of the estimates, and involved CS recovery problems may become intractable.

We consider compressed acquisition and progressive reconstruction of spatio-temporal signals in multi-hop WSNs. We propose a novel, sliding window based CS method allowing for instantaneous recovery of WSN readings from periodically collected measurements. The method uses previously decoded estimates to progressively refine the estimates, and contrary to the works above, can control decoding delay and complexity. A numerical example shows that our proposed method leads to higher reconstruction accuracy, yet with lower decoding delay and complexity, as compared to the state of the art methods.

II. Network Model

Consider a single-sink multi-hop WSN with set of sensors $\mathcal{N} = \{1, \dots, N\}$. They observe a phenomenon and record T data samples in each sensing period. The readings are assumed to have spatial and temporal dependency, illustrated in Fig. 1. The sensors communicate with fixed multi-hop routing under appropriate medium access control and pre-defined scheduling.

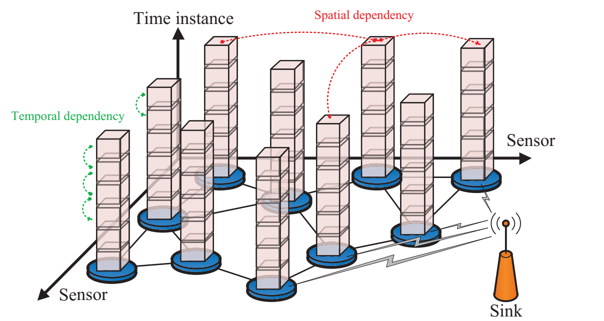

Fig. 1. A wireless sensor network with temporally and spatially correlated data samples with $N = 10$ and $T = 6$.

III. Compressed Sensing of Correlated Data

Let $\boldsymbol{X} \in \mathbb{R}^{T \times N}$ denote the WSN signal ensemble. Thus, its (t, i)th entry, x_{ti}, represents tth data sample of sensor $i \in \mathcal{N}$, column $\boldsymbol{x}_i \in \mathbb{R}^T$ ith sensor's data and row $\boldsymbol{x}^t \in \mathbb{R}^N$ the readings at time instance t. We assume that $\boldsymbol{X}$ has compressible representations in proper bases, denoted as $\boldsymbol{\Psi}_\mathrm{T} \in \mathbb{R}^{T \times T}$ and $\boldsymbol{\Psi}_\mathrm{S} \in \mathbb{R}^{N \times N}$ for temporal and spatial domain, respectively. Thus, $\boldsymbol{x}^t = \boldsymbol{\Psi}_\mathrm{S}\boldsymbol{\theta}_{\mathrm{S},t}$, where $\boldsymbol{\theta}_{\mathrm{S},t} \in \mathbb{R}^N$ are $K_{\mathrm{S},t}$-compressible spatial domain coefficients and $\boldsymbol{x}_i = \boldsymbol{\Psi}_\mathrm{T}\boldsymbol{\theta}_{\mathrm{T},i}$, where $\boldsymbol{\theta}_{\mathrm{T},i} \in \mathbb{R}^T$ are $K_{\mathrm{T},i}$-compressible temporal domain coefficients.

Kronecker sparsifying bases are able to combine different correlation patterns from each signal dimension into a single matrix [11]. Thus, we can merge the transformations above as

$$\boldsymbol{x} = \mathrm{vec}(\boldsymbol{X}^\mathrm{T}) = (\boldsymbol{\Psi}_\mathrm{T} \otimes \boldsymbol{\Psi}_\mathrm{S})\mathrm{vec}(\boldsymbol{Z}) = \boldsymbol{\Psi}\boldsymbol{z}, \tag{1}$$

where $\boldsymbol{x} = [(\boldsymbol{x}^1)^\mathrm{T}, \dots, (\boldsymbol{x}^T)^\mathrm{T}]^\mathrm{T} \in \mathbb{R}^{TN}$, $\otimes$ denotes the Kronecker product, $\boldsymbol{\Psi} = (\boldsymbol{\Psi}_\mathrm{T} \otimes \boldsymbol{\Psi}_\mathrm{S}) \in \mathbb{R}^{TN \times TN}$ is the Kronecker sparsifying basis, and $\boldsymbol{z} = \mathrm{vec}(\boldsymbol{Z}) \in \mathbb{R}^{TN}$ are the K_J-compressible joint transform domain coefficients.

At each time instance, $J_t < N$ CS measurements $\boldsymbol{v}_t \in \mathbb{R}^{J_t}$ are acquired via measurement matrix $\boldsymbol{\Omega}_t \in \mathbb{R}^{J_t \times N}$ as

$$\boldsymbol{v}_t = \boldsymbol{\Omega}_t \boldsymbol{x}^t, \ t = 1, \dots, T. \tag{2}$$

Conventional CS decoding recovers each $\boldsymbol{x}^t$ separately from (2) as $\hat{\boldsymbol{\theta}}_{\mathrm{S},t} := \arg\min_{\boldsymbol{\theta}} \|\boldsymbol{\theta}\|_1$ s.t. $\boldsymbol{v}_t = \boldsymbol{\Omega}_t\boldsymbol{\Psi}_\mathrm{S}\boldsymbol{\theta}$. By (1), signal $\boldsymbol{X}$ can be jointly recovered from $\boldsymbol{v} = [\boldsymbol{v}_1^\mathrm{T}, \dots, \boldsymbol{v}_T^\mathrm{T}]^\mathrm{T} \in \mathbb{R}^{\bar{J}}$ as

$$\hat{\boldsymbol{z}} := \arg\min_{\boldsymbol{z}} \|\boldsymbol{z}\|_1 \quad \text{s.t.} \quad \boldsymbol{v} = \boldsymbol{\Omega}\boldsymbol{\Psi}\boldsymbol{z}, \tag{3}$$

where $\boldsymbol{\Omega} = \text{diag}\{\boldsymbol{\Omega}_1, \ldots, \boldsymbol{\Omega}_T\} \in \mathbb{R}^{\bar{J} \times NT}$ and $\bar{J} = \sum_{t=1}^{T} J_t$.

IV. Progressive Joint Signal Recovery

We will derive a novel joint data acquisition and reconstruction method which uses sliding window signal recovery process to control decoding delay and complexity. The method is able to instantaneously reconstruct WSN samples and also to refine the estimates in successive decoding instances.

Let $\boldsymbol{X}_{\text{W}}^t = [(\boldsymbol{X}_{\text{B}}^t)^{\text{T}}, \boldsymbol{x}^t]^{\text{T}} \in \mathbb{R}^{W_t \times N}$ denote the data block inside the sliding window of size $W_t > 0$ at time instance $t = 1, \ldots, T$, where $\boldsymbol{X}_{\text{B}}^t = [\boldsymbol{x}^{t-D_t}, \ldots, \boldsymbol{x}^{t-1}]^{\text{T}} \in \mathbb{R}^{D_t \times N}$ represents the samples of $D_t = W_t - 1$ previous time instances. Similar to (1), $\boldsymbol{X}_{\text{W}}^t$ has a compressible representation as

$$\boldsymbol{x}_{\text{W}}^t = \text{vec}\big([\boldsymbol{X}_{\text{W}}^t]^{\text{T}}\big) = (\boldsymbol{\Psi}_{\text{T}_{\text{W}}} \otimes \boldsymbol{\Psi}_{\text{S}})\boldsymbol{z}_{\text{W}}^t = \boldsymbol{\Psi}_{\text{W}} \boldsymbol{z}_{\text{W}}^t, \quad (4)$$

where $\boldsymbol{x}_{\text{W}}^t = [(\boldsymbol{x}^{t-D_t})^{\text{T}}, \ldots, (\boldsymbol{x}^t)^{\text{T}}]^{\text{T}} \in \mathbb{R}^{NW_t}$, $\boldsymbol{\Psi}_{\text{T}_{\text{W}}} \in \mathbb{R}^{W_t \times W_t}$ is the temporal domain basis, $\boldsymbol{\Psi}_{\text{W}} \in \mathbb{R}^{NW_t \times NW_t}$ is the Kronecker sparsifying basis and $\boldsymbol{z}_{\text{W}}^t \in \mathbb{R}^{NW_t}$ are K_{W}^t-compressible joint transformation coefficients. From (2), the CS measurements, $\boldsymbol{v}_{\text{W}}^t = [\boldsymbol{v}_{t-D_t}^{\text{T}}, \ldots, \boldsymbol{v}_t^{\text{T}}]^{\text{T}} \in \mathbb{R}^{\bar{J}_t}$, reads as

$$\boldsymbol{v}_{\text{W}}^t = \boldsymbol{\Omega}_{\text{W}}^t \boldsymbol{x}_{\text{W}}^t, \ t = 1, \ldots, T \quad (5)$$

where $\boldsymbol{\Omega}_{\text{W}}^t = \text{diag}\{\boldsymbol{\Omega}_{t-D_t}, \ldots, \boldsymbol{\Omega}_t\} \in \mathbb{R}^{\bar{J}_t \times NW_t}$ and $\bar{J}_t = \sum_{\tau=t-D_t}^{t} J_\tau$. By (3), $\boldsymbol{X}_{\text{W}}^t$ can be recovered from

$$\hat{\boldsymbol{z}}_{\text{W}}^t := \arg\min_{\boldsymbol{z}} \|\boldsymbol{z}\|_1 \quad \text{s.t.} \quad \boldsymbol{v}_{\text{W}}^t = \boldsymbol{\Omega}_{\text{W}}^t \boldsymbol{\Psi}_{\text{W}} \boldsymbol{z}. \quad (6)$$

Note that, in general, each $\boldsymbol{x}^t$ is reconstructed several times in (6). Thus, the resulting estimates can be used as prior information in subsequent recovery instances, discussed next.

Let us fix $W_t = W$, $t \geq W$, when $D_t = D$. At time instance $t > W$, the estimates of WSN readings from D previous time instances are stored in decoder buffer as $\boldsymbol{b}^t = \hat{\boldsymbol{x}}_{\text{B},(t-1)}^t = [(\hat{\boldsymbol{x}}_{(t-1)}^{t-D})^{\text{T}}, \ldots, (\hat{\boldsymbol{x}}_{(t-1)}^{t-1})^{\text{T}}]^{\text{T}}$, where $\hat{\boldsymbol{x}}_{(t-1)}^{t-d} \in \mathbb{R}^N$ is the estimate of $\boldsymbol{x}^{t-d}$ obtained at decoding instance $t-1$. We will utilize these estimates to recover $\boldsymbol{X}_{\text{W}}^t = [(\boldsymbol{X}_{\text{B}}^t)^{\text{T}}, \boldsymbol{x}^t]^{\text{T}}$ by modifying problem (6) with an added regularization term as

$$\text{min.} \ \|\boldsymbol{z}\|_1 + \epsilon_{\text{B}} \big\|\boldsymbol{\Psi}_{\text{B}} \boldsymbol{z} - \boldsymbol{b}^t\big\|_2 \quad \text{s.t.} \quad \boldsymbol{v}_{\text{W}}^t = \boldsymbol{\Omega}_{\text{W}}^t \boldsymbol{\Psi}_{\text{W}} \boldsymbol{z}, \ (7)$$

where $\boldsymbol{z} = [z_1, \ldots, z_{NW}]^{\text{T}}$ are the optimization variables, $\epsilon_{\text{B}} \geq 0$ is a weighting parameter and matrix $\boldsymbol{\Psi}_{\text{B}} \in \mathbb{R}^{ND \times NW}$ consists of the first ND rows of basis $\boldsymbol{\Psi}_{\text{W}}$ given in (4).

With obtained variables, the regularization term in (7) becomes $\epsilon_{\text{B}} \big\|\boldsymbol{\Psi}_{\text{B}} \hat{\boldsymbol{z}}_{\text{W}}^t - \boldsymbol{b}^t\big\|_2$, i.e., $\epsilon_{\text{B}} \big\|\text{vec}\big(\hat{\boldsymbol{X}}_{\text{B},(t)}^t - \hat{\boldsymbol{X}}_{\text{B},(t-1)}^t\big)\big\|_2$. Thus, it induces additional penalty (w.r.t. L_2-norm) by the deviation of the estimates of $\boldsymbol{X}_{\text{B}}^t$ obtained at consecutive decoding instances. The emphasis between this deviation and the sparsity-encouraging L_1-norm term is controlled by ϵ_{B}.

V. Numerical Example

We considered a WSN with $N = 16$ and $T = 60$, in which the data correlation was created similarly as in [8]. We evaluate our proposed method (Prog-CS) in terms of reconstruction error achieved with given number of measurements $J = J_t$, $\forall t = 1, \ldots, T$. Two considered benchmark methods were 1) Kronecker CS (Kron-CS) with joint reconstruction in (3), and 2) Spatial CS (Spat-CS) with separate recovery for each $\boldsymbol{x}^t$.

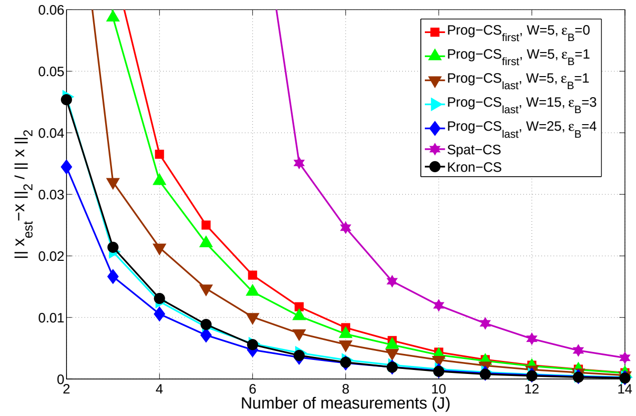

Fig. 2. CS recovery performance of Prog-CS, Kron-CS and Spat-CS.

Fig. 2 depicts the average CS recovery error against the number of measurements. Prog-CS_{first} uses instantaneous estimates whereas Prog-CS_{last} the ones refined D times. Firstly, it can be seen that by incorporating the prior information on estimates ($\epsilon > 0$), Prog-CS_{first} can slightly improve the performance of that without the memory ($\epsilon = 0$). This becomes more evident in Prog-CS_{last}: with $W = 15$, Prog-CS_{last} matches the performance of Kron-CS, and with $W = 25$, it can even outperform Kron-CS. Since Prog-CS and Kron-CS exploit the joint signal dependency, they both clearly outperform Spat-CS. In conclusion, Prog-CS achieved the same CS recovery performance as the state of the art Kron-CS, yet with reduced decoding delay and complexity of factor $W/T = 1/4$.

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