On Minimum-Time Scheduling Problem in a Class of Wireless Networks

Extended Abstract

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I. Introduction

We consider a set of transmitter-receiver pairs, or links, that share a wireless medium, and address the problem of emptying backlogged queues with given initial size at the transmitters in minimum time. The minimum-time scheduling problem (MTSP) amounts to determining activation subsets of links, and their time durations, to form a minimum-time schedule. This scheduling problem has been studied a great deal in the past, either through the classical protocol model, or through the current physical model, which enables a cross-layered view that combines rate and power control with overall network resource allocation.

Assume a wireless network with N links, each link i has a finite amount of bits d_i to be transmitted. Let $\mathcal{H}$ denote the union of all subsets of $\mathcal{N}$, excluding the empty set. Clearly, $|\mathcal{H}| = 2^N - 1$. We use the term *group* to refer to a member $\mathcal{c} \in \mathcal{H}$. Scheduling a group $\mathcal{c}$ of the N links means that all links in $\mathcal{c}$ are concurrently activated, with a given fixed power, for some positive amount of time $t_{\mathcal{c}}$. For any group $\mathcal{c}$, the effective transmit rate $r_{i\mathcal{c}}$ of any link $i \in \mathcal{c}$ directly depends on the composition of the activation group. If all the link rates for all $2^N - 1$ groups $\mathcal{c} \in \mathcal{H}$ are known, the MTSP accepts the following linear programming (LP) formulation:

$$\min \quad \sum_{\mathcal{c} \in \mathcal{H}} t_{\mathcal{c}} \tag{1a}$$

$$\text{s. t.} \quad \sum_{\mathcal{c} \in \mathcal{H}} r_{i\mathcal{c}} t_{\mathcal{c}} = d_i \quad i = 1, \ldots, N \tag{1b}$$

$$\boldsymbol{t} \geq 0 \tag{1c}$$

The MTSP has been proved to be $\mathcal{NP}$-hard for both discrete and continuous link rates (e.g., [1], [2]).

II. Polynomial Minimum-Time Scheduling Problem in Structured Wireless Networks

We consider the MTSP in a class of structured wireless networks in which the links can be divided into K, where $K \leq N$, clusters and have the following properties:

- The links in the same cluster have the same rate if they are activated.
- The rate value of a link is fully determined by the number of activated links in every cluster and the cluster that the link belongs to, but not on the identity of the links.

We refer to it as Multi-Cluster Cardinality-based Rates (MCCR). An example scenario corresponding to this case consists in transmitters that are co-located at a central point, resembling a system with multiple transmitter base station, and receivers are distributed on several circles around it. Then the receivers on same circle have equal distance to their transmitters and hence identical geometric channel gain. A special case in this problem class with $K = 1$, a.k.a., cardinality-based rate, has been considered and investigated in [1] and [3].

III. Contributions

First, we provide fundamental insights on the MTSP with MCCR, showing any instance of it can be solved in polynomial time; and presenting the optimality conditions for problem decomposition with uniform or arbitrary demands. Second, we generalize the MCCR model to more practical networks by partitioning the randomly located users into several clusters with k-means clustering. A column generation algorithm is proposed to solve the problem effectively. Besides approximating the global optimal solution, the approach servers well for obtaining lower and upper bounds of optimal schedule length, which can be utilized to evaluate solutions of heuristic algorithms. We have performed the proposed setup in both small and medium-sized wireless networks. Numerical results show that, the approach leads to a satisfactory result even when K is set to a relatively small number in relation to N. The optimality gap of no more than 4.0 percent on average can be reached with low computational complexity.

References

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