Transmission Power Variance Constrained Power Allocation for Iterative Frequency Domain Multiuser SIMO Detector

Valtteri Tervo*+, A Tölli*, J. Karjalainen†, Tad Matsumoto*+

{wade, atolli, matumoto}@ee.oulu.fi, juha.karjalainen@renesasmobile.com

*Centre for Wireless Communications, University of Oulu, P.O. Box 4500, 90014 University of Oulu, Finland.

+Japan Advanced Institute of Science and Technology, 1-1 Asahi-Dai, Nomi, Ishikawa, 923-1292 Japan.

†Renesas Mobile Europe Oy, Elektroniikkatie 13, 90590 OULU, Finland

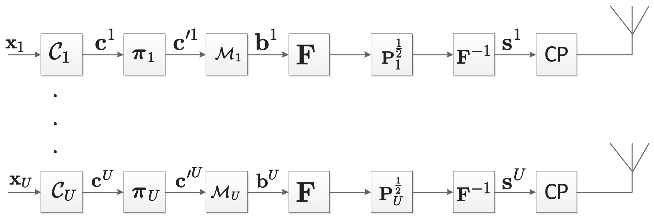

Fig. 1. The block diagram of the transmitter side of the system model.

***Abstract*—Transmission power variance constrained power allocation in single carrier multiuser (MU) single-input multiple-output (SIMO) systems with iterative frequency domain (FD) soft cancelation (SC) minimum mean squared error (MMSE) equalization is considered in this work. It is known in the literature that peak to average power ratio (PAPR) at the transmitter can be decreased by reducing the variance of the transmit power. In this research, we derive a power variance constraint to statistically control the PAPR. This constraint is plugged into a convergence constrained power allocation (CCPA) problem and successive convex approximation (SCA) approach via series of geometric programs (GP) is developed. Numerical results are presented in the form of complementary cumulative distribution functions (CCDFs) to demonstrate the effectiveness of the proposed method.**

I. System Model and Constraints

Consider a single carrier uplink transmission with U single-antenna users and a base station with N_R antennas as depicted in Fig. 1. Due to the lack of space we guide the reader to check [1], [2] for more detailed system model description.

Instead of considering the instant PAPR, we will derive the expected variance of the transmit power which is not depending on the instantaneous symbol sequence. Because the power variance is derived similarly for all the users, the user index is omitted in this section. Due to the lack of space the intermediate steps are not shown in this abstract. The expected variance of the transmit power is shown to be

$$\Sigma^2(\mathbf{P}) = \frac{N_F-1}{N_F^3}(\sum_{l=1}^{N_F} P_l)^2 - \frac{1}{N_F^3}\sum_{p,q\in\mathcal{S}_1}^{N_F} P_pP_q - \frac{1}{N_F^3}\sum_{p,q,r,s\in\mathcal{S}_2}^{N_F}\sqrt{P_pP_qP_rP_s}, \quad (1)$$

where $\mathcal{S}_1 = \{p,q \in \{1,2,\dots,N_F\} : p \neq q, p-q = \pm N_F/2\}$ and $\mathcal{S}_2 = \{p,q,r,s \in \{1,2,\dots,N_F\} : p \neq q, r \neq s, (p,q) \neq (r,s), s-r \in \{p-q, N_F+p-q, -N_F+p-q\}\}$. The objective is to control the variance of the normalized power and hence P_l in (1) is divided by $\sum_{n=1}^{N_F} P_n, \forall l$, resulting

$$\Sigma^2(\mathbf{P}) \leq \sigma_s^2(\sum_{l=1}^{N_F} P_l)^2, \quad (2)$$

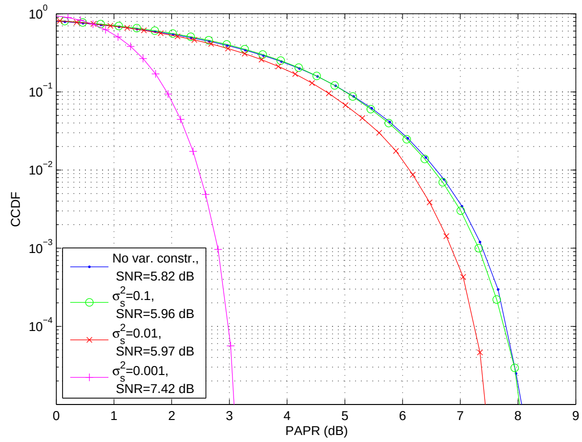

Fig. 2. CCDF of PAPR for user 2. $U = 2$, $N_F = 8$, $N_R = 2$, bit error probability target is 10^{-5} for all users, QPSK with Gray mapping, and systematic repeat accumulate (RA) code [3] with a code rate 1/3 and 8 internal iterations are used. The signal-to-noise ratio per receiver antenna averaged over frequency bins is defined by SNR= $\mathrm{tr}\{\mathbf{P}\}/(N_R N_F \sigma_v^2)$. The channel we consider is a quasi-static Rayleigh fading 5-path average equal gain channel.

where $\sigma_s^2 \in \mathbb{R}^+$ is the maximum variance of the normalized power. Plugging (1) to (2) the constraint can be written as

$$(N_F-1)(\sum_{l=1}^{N_F} P_l)^2 \leq \sum_{p,q\in\mathcal{S}_1}^{N_F} P_pP_q + \sum_{p,q,r,s\in\mathcal{S}_2}^{N_F}\sqrt{P_pP_qP_rP_s} + (\sum_{l=1}^{N_F} P_l)^2\sigma_s^2 N_F^3. \quad (3)$$

The convergence constraint of an iterative equalizer is written as [1]

$$\zeta_{u,k} \geq \xi_{u,k}, \forall u = 1,2\dots,U, \forall k = 1,2,\dots,K, \quad (4)$$

where $\zeta_{u,k}$ is the effective signal-to-interference-plus-noise ratio (SINR) of the prior symbol estimates and $\xi_{u,k}$ is a constant depending on the forward error correction (FEC) code.

II. Successive Convex Approximation

Our objective is to minimize the total transmitted power with the constraints (3) and (4). The objective is linear but both (3) and (4), are nonconvex constraints. However, we can derive a successive convex approximation for the problem via geometric program (GP) [4] using the inequality [1]

$$\sum_{m=1}^{N_F} t_m \geq \prod_{m=1}^{N_F}(\frac{t_m}{\Phi_m})^{\Phi_m}, \quad (7)$$

where $\Phi_m = \frac{\hat{t}_m}{\sum_{n=1}^{N_F}\hat{t}_n}$, $\hat{t}_m > 0$, and $t_m > 0$, $m = 1,2,\dots,N_F$.

$$\mathcal{A}_u(\mathbf{P}_u) = \left(\frac{\prod_{p,q\in\mathcal{S}_1} \left(\frac{P_{u,p}P_{u,q}}{\theta^{(1)}_{u,pq}} \right)^{\theta^{(1)}_{u,pq}}}{\tau^{(1)}_u} \right)^{\tau^{(1)}_u} \left(\frac{\prod_{p,q,r,s\in\mathcal{S}_2} \left(\frac{\sqrt{P_{u,p}P_{u,q}P_{u,r}P_{u,s}}}{\theta^{(2)}_{u,pqrs}} \right)^{\theta^{(2)}_{u,pqrs}}}{\tau^{(2)}_u} \right)^{\tau^{(2)}_u}$$
$$\times \left(\frac{\sigma^2 N_F^3 \prod_{l=1}^{N_F} \left(\frac{P^2_{u,l}}{\theta^{(3)}_{u,l}} \right)^{\theta^{(3)}_{u,l}}}{\tau^{(3)}_u} \right)^{\tau^{(3)}_u} \left(\frac{2\sigma_s^2 N_F^3 \prod_{\substack{p,q=1\\q>p}}^{N_F} \left(\frac{P_{u,p}P_{u,q}}{\theta^{(4)}_{u,pq}} \right)^{\theta^{(4)}_{u,pq}}}{\tau^{(4)}_u} \right)^{\tau^{(4)}_u} \quad (5)$$

$$\tau^{(1)}_u = \frac{\sum_{p,q\in\mathcal{S}_1} P_{u,p}P_{u,q}}{\sum_{p,q\in\mathcal{S}_1} P_{u,p}P_{u,q} + \sum_{p,q,r,s\in\mathcal{S}_2} \sqrt{P_{u,p}P_{u,q}P_{u,r}P_{u,s}} + (\sum_{l=1}^{N_F} P_{u,l})^2 \sigma_s^2 N_F^3}$$
$$\tau^{(2)}_u = \frac{\sum_{p,q,r,s\in\mathcal{S}_2} \sqrt{P_{u,p}P_{u,q}P_{u,r}P_{u,s}}}{\sum_{p,q\in\mathcal{S}_1} P_{u,p}P_{u,q} + \sum_{p,q,r,s\in\mathcal{S}_2} \sqrt{P_{u,p}P_{u,q}P_{u,r}P_{u,s}} + (\sum_{l=1}^{N_F} P_{u,l})^2 \sigma_s^2 N_F^3}$$
$$\tau^{(3)}_3 = \frac{\sigma_s^2 N_F^3 \sum_{l=1}^{N_F} P^2_{u,l}}{\sum_{p,q\in\mathcal{S}_1} P_{u,p}P_{u,q} + \sum_{p,q,r,s\in\mathcal{S}_2} \sqrt{P_{u,p}P_{u,q}P_{u,r}P_{u,s}} + (\sum_{l=1}^{N_F} P_{u,l})^2 \sigma_s^2 N_F^3}$$
$$\tau^{(4)}_u = \frac{2\sigma_s^2 N_F^3 \sum_{\substack{p,q=1\\q>p}}^{N_F} P_{u,p}P_{u,q}}{\sum_{p,q\in\mathcal{S}_1} P_{u,p}P_{u,q} + \sum_{p,q,r,s\in\mathcal{S}_2} \sqrt{P_{u,p}P_{u,q}P_{u,r}P_{u,s}} + (\sum_{l=1}^{N_F} P_{u,l})^2 \sigma_s^2 N_F^3}. \quad (6)$$

Applying (7) twice to (3) and once to (4), a successive convex approximation of both the convergence and power variance constrained power minimization problem can be written as

$$\begin{aligned}
\underset{\mathbf{P},\mathbf{t}}{\text{minimize}} \quad & \text{tr}\{\mathbf{P}\} \\
\text{subject to} \quad & \textstyle\prod_{n=1}^{N_F} (\frac{t^k_{u,n}}{\Phi^k_{u,n}})^{\Phi^k_{u,n}} \geq N_F \xi_{u,k}, \\
& u = 1,2,\ldots,U, k = 1,2,\ldots,K, \\
& P_{u,m} |\boldsymbol{\omega}^{k}_{u,m}{}^{\mathrm{H}} \boldsymbol{\gamma}_{u,m}|^2 \geq \\
& (\textstyle\sum_{l=1}^{U} P_{l,m} |\boldsymbol{\omega}^{k}_{u,m}{}^{\mathrm{H}} \boldsymbol{\gamma}_{l,m}|^2 \bar{\Delta}_{l,k} + \sigma^2 |\boldsymbol{\omega}^k_{u,m}|^2) t^k_{u,m}, \\
& u = 1,2,\ldots,U, k = 1,2,\ldots,K, \\
& m = 1,2,\ldots,N_F, \\
& (N_F - 1)(\textstyle\sum_{l=1}^{N_F} P_{u,l})^2 \leq \mathcal{A}_u(\mathbf{P}_u), u = 1,2,\ldots,U, \\
& P_{u,m} \geq 0, \quad u = 1,2,\ldots,U, m = 1,2,\ldots,N_F,
\end{aligned} \quad (8)$$

where $\mathcal{A}_u(\mathbf{P}_u)$ is given in (5) and (6) and

$$\begin{aligned}
\theta^{(1)}_{u,pq} &= \frac{P_{u,p}P_{u,q}}{\sum_{p',q'\in\mathcal{S}_1} P_{u,p'}P_{u,q'}}, \\
\theta^{(2)}_{u,pqrs} &= \frac{\sqrt{P_{u,p}P_{u,q}P_{u,r}P_{u,s}}}{\sum_{p',q',r',s'\in\mathcal{S}_2} \sqrt{P_{u,p'}P_{u,q'}P_{u,r'}P_{u,s'}}}, \\
\theta^{(3)}_{u,l} &= \frac{P^2_{u,l}}{\sum_{l'=1}^{N_F} P^2_{u,l'}}, \theta^{(4)}_{u,l} = \frac{P_{u,p}P_{u,q}}{\sum_{\substack{p',q'=1\\q'>p'}}^{N_F} P_{u,p'}P_{u,q'}}.
\end{aligned} \quad (9)$$

The SCA algorithm is summarized in **Algorithm** 1, where the superscript * denotes the optimal solution of (8).

Algorithm 1 Successive convex approximation algorithm.

1: Set $\hat{t}^k_{u,n} = \hat{t}^{k(0)}_{u,n}, \forall u,k,n$ and $\hat{\mathbf{P}}_{u,n} = \hat{\mathbf{P}}^{(0)}_{u,n}, \forall u,n.$
2: **repeat**
3: Calculate the weights (6) and (9).
4: Solve Eq. (8).
5: Update $\hat{t}^k_{u,n} = t^{k(*)}_{u,n}, \forall u,k,n$ and $\hat{\mathbf{P}}_{u,n} = \hat{\mathbf{P}}^{(*)}_{u,n}, \forall u,n.$
6: **until** Convergence.

III. Numerical Results

The joint optimum of transmitter and receiver can be achieved via alternating optimization [1] which means that the problem is split to the optimization of transmit power for fixed receiver and optimization of receiver for fixed power allocation. The transmitter optimization step is performed using **Algorithm** 1 and the optimum receiver is given in [1].

The complementary cumulative distribution function (CCDF) of PAPR for user 2 for different values of σ_s^2 is depicted in Fig. 2. It can be seen from the Fig. 2 that when $\sigma_s^2 = 0.1$ there is not much difference compared to the case where there is no variance constraint. When $\sigma_s^2 = 0.01$ we can obtain a slight PAPR gain with roughly the same SNR compared to the case with no variance constraint. When σ_s^2 is further reduced to 0.001 the PAPR gain is significant. Even though the required SNR to achieve the target MI point increases 1.6 dB, the PAPR gain is much larger than the SNR loss. For example, in the case of no normalized variance constraint we may need to set the maximum transmission power according to 8 dB PAPR while in the case of $\sigma_s^2 = 0.001$ the PAPR corresponding the same value of CCDF ($10^{-4.74}$) is 3.06 dB. Hence, for $10^{-4.74}$ outage the gain is 8 dB - 3.06 dB - 1.6 dB = 3.34 dB. Therefore, the coverage of $\sigma_s^2 = 0.001$ precoded transmission is larger compared to the case with no variance constraint.

IV. Conclusions

Transmission power variance constrained power allocation for iterative frequency domain multiuser single input multiple output detector was derived in this work. The precoding technique takes into account the convergence properties of the iterative receiver while keeping the transmission power variance below the desired threshold. Successive convex approximation (SCA) approach via series of geometric programs (GP) is developed. Numerical results demonstrated that the PAPR gain is significantly larger than the SNR loss in the variance constrained precoding technique compared to the case without variance constraint. Hence, the proposed precoding technique increases the coverage of the transmission and is beneficial for power limited cell edge users.

References

[1] V. Tervo, A. Tölli, J. Karjalainen, and T. Matsumoto, "Convergence constrained multiuser transmitter-receiver optimization in single carrier FDMA," *IEEE Trans. Signal Processing*, 2013, (under review), arXiv:1310.8067.

[2] ——, "PAPR constrained power allocation for iterative frequency domain multiuser SIMO detector," in *Proc. IEEE Int. Conf. Commun.*, Sydney, Australia, Jun.10–14 2014, pp. 1–6 (submitted), arXiv:1311.5376.

[3] D. Divsalar, H. Jin, and R. J. McEliece, "Coding theorems for 'turbo-like' codes," in *Proc. Annual Allerton Conf. Commun., Contr., Computing*, Urbana, Illinois, USA, Sep.23–25 1998, pp. 201–210.

[4] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.