Equilibrium Models of Wireless Networks with Moving Users

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***Abstract*—We consider a general problem of evaluation of a stable flows distribution in a wireless communication network with moving nodes. By using approximations of frequency values for nodes and equilibrium conditions we formulate the above problem as a non-stationary variational inequality which also reduces to an optimization one. By solving each approximate problem within some tolerances we create a sequence tending to a solution of the desired "limit" problem.**

***Index Terms*—Communication wireless networks, moving nodes, flows distribution, equilibrium approach, non-stationary optimization, iterative methods.**

Recent development of wireless telecommunication networks provides many new information services for users together with wide availability and better information quality. However, efficient management of such networks meets a lot of new problems due to their specific features such as the absence of predefined physical links between nodes, mobility of nodes, and boundedness of the batteries capacity, which are different from the usual wire communication networks; see, e.g., [1]. Because of the stochastic character of nodes movement and absence of any central decision maker, the most solution methods for such problems are mostly heuristic, hence their substantiation is deduced from simulation procedures. In this talk, we follow the equilibrium approach based on the corresponding mathematical model for creation of more efficient network control decisions. We treat a problem of flows distribution in a communication network with moving nodes as a non-stationary variational inequality via a suitable equilibrium approach. This approach can be viewed as further development of that from [2], where each moving (user) node was treated as an independent Markovian chain. It enables us to cover more general classes of applications. Next, by solving each approximate problem within some tolerances we create a sequence of flows tending to a solution to the desired problem.

The model is determined on a network involving a finite set of nodes (users) $\mathcal{N}$, who are distributed within a region $\mathcal{R}$, which is supposed to be rectangular for simplicity. That is, $\mathcal{R}$ is represented by an $m \times n$ matrix, whose elements are equal square cells c_{ij} for $i = 1, \dots, m$ and $j = 1, \dots, n$. Next, we select a subset of origin-destination (O/D) pairs $\mathcal{W}$ that among all the pairs of users, i.e. $\mathcal{W} \subseteq \mathcal{N} \times \mathcal{N}$, and each (O/D) pair $w \in \mathcal{W}$ is associated with a non-negative flow demand b_w. Next, time is supposed to be divided into equal slots $t = 1, 2, \dots$, all the users can move within the region $\mathcal{R}$, but only one transition (cell change) is possible during one time slot. The current state of user i at time slot t is described by the matrix

$$P^{(i,t)} = \left(\pi_{kl}^{(i,t)}\right) \tag{1}$$

where $\pi_{k,l}^{(i,t)}$ denotes the current frequency probability for the i-th user to be in cell c_{kl} at time slot t for $k = 1, \dots, m$ and $l = 1, \dots, n$. Hence, if the i-th user was $s(t)$ times in cell c_{kl} for time slots $k = 1, 2, \dots, t$, then $\pi_{k,l}^{(i,t)} = s(t)/t$. Clearly,

$$\sum_{k=1}^{m}\sum_{l=1}^{n} \pi_{kl}^{(i,t)} = 1$$

for any i and t. The initial matrix $P^{(i,1)}$ can be defined as follows

$$\pi_{kl}^{(i,1)} = \begin{cases} 1 & \text{if the } i\text{-th user is in cell } c_{kl}, \\ 0 & \text{otherwise.} \end{cases}$$

Assume that the sequence of matrices $\left\{P^{(i,t)}\right\}$ tends to some stable (limit) probability distribution matrix

$$\bar{P}^{(i)} = \left(\bar{\pi}_{kl}^{(i)}\right)$$

as $t \to \infty$ for each $i \in \mathcal{N}$.

We suggest to take the (limit) stationary probabilities for creating the network equilibrium problem. We intend to describe behavior of the system within the stationary trajectory part. Given the matrix $\bar{P}^{(i)}$, we introduce a distance threshold γ and then for each pair of nodes i, j we calculate the mean distance

$$u_{ij} = \sum_{k=1}^{m}\sum_{l=1}^{n}\sum_{\kappa=1}^{m}\sum_{\lambda=1}^{n} \bar{\pi}_{kl}^{(i)} \bar{\pi}_{\kappa\lambda}^{(j)} \rho((k,l),(\kappa,\lambda)),$$

where $\rho((k,l),(\kappa,\lambda))$ denotes the distance between the centers of cells (k,l) and (κ,λ). Hence, if $u_{ij} \leq \gamma$, then arc $a = (i,j)$ is included in the set of communication links $\bar{\mathcal{A}}$.

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In such a way we determine the path-arc incidence matrix $\bar{A}$ with the elements

$$\bar{\alpha}_{pa} = \begin{cases} 1 & \text{if arc } a \text{ belongs to path } p, \\ 0 & \text{otherwise;} \end{cases}$$

and, for any selected (O/D) pair $w \in \mathcal{W}$ we can define the set of paths $\bar{\mathcal{P}}_w$ joining this pair. Then the feasible set of network path flows is defined as follows:

$$\bar{X} = \left\{ x \;\middle|\; \sum_{p \in \bar{\mathcal{P}}_w} x_p = b_w, x_p \geq 0,\; p \in \bar{\mathcal{P}}_w, w \in \mathcal{W} \right\}. \quad (2)$$

Next, we determine the value of the arc flow

$$f_a = \sum_{w \in \mathcal{W}} \sum_{p \in \bar{\mathcal{P}}_w} \bar{\alpha}_{pa} x_p \quad (3)$$

for each arc $a \in \bar{\mathcal{A}}$. We assume that for each link $a = (i, j)$ and for each pair of cells (k, l) and (κ, λ) we know a continuous function $T_a^{\{(k,l),(\kappa,\lambda)\}}$, whose value $T_a^{\{(k,l),(\kappa,\lambda)\}}(f_a)$ gives the delay in traversing link a with one unit of flow when user i is situated in cell (k, l), user j is situated in cell (κ, λ), and the flow value on this arc is f_a. The summary cost at path flows vector x for path p has the form:

$$\bar{G}_p(x) = \sum_{k=1}^{m} \sum_{l=1}^{n} \sum_{\kappa=1}^{m} \sum_{\lambda=1}^{n} \left(\sum_{a=(i,j) \in \bar{\mathcal{A}}} \bar{\pi}_{kl}^{(i)} \bar{\pi}_{\kappa\lambda}^{(j)} \bar{\alpha}_{pa} T_a^{\{(k,l),(\kappa,\lambda)\}}(f_a) \right).$$

The network equilibrium problem is known to be rewritten as a variational inequality problem (VI for short); see e.g. [3], [4]. More precisely, we now utilize the following VI: Find $x^* \in \bar{X}$ such that

$$\langle \bar{G}(x^*), x - x^* \rangle = \sum_{w \in \mathcal{W}} \sum_{p \in \bar{\mathcal{P}}_w} \bar{G}_p(x^*)(x_p - x_p^*) \geq 0 \quad \forall x \in \bar{X}. \quad (4)$$

Besides, due to the integrability of T_a, we can also associate (2)–(4) with a suitable optimization problem.

However, the limit matrix $\bar{P}^{(i)}$ remains unknown for us and we can not solve problem (2)–(4) directly. Nevertheless, we can utilize its approximation at time slot t by using the matrix $P^{(i,t)}$ in (1) for $t = 1, 2, \ldots$ The corresponding online approximation of the "limit" network equilibrium problem (4) at time slot t is also written in the form of VI: Find $x^{(t,*)} \in X^{(t)}$ such that

$$\langle G^{(t)}(x^{(t,*)}), x^{(t)} - x^{(t,*)} \rangle = \sum_{w \in \mathcal{W}} \sum_{p \in \mathcal{P}_w^{(t)}} G_p^{(t)}(x^*)(x_p^{(t)} - x_p^{(t,*)}) \geq 0 \quad (5)$$
$$\forall x^{(t)} \in X^{(t)};$$

where $X^{(t)}$, $G^{(t)}$, and $\mathcal{P}_w^{(t)}$ are proper approximations of $\bar{X}$, $\bar{G}$, and $\bar{\mathcal{P}}_w$, respectively.

Thus, instead of the basic limit VI (4) we have a sequence of its approximations of form (5) for $t = 1, 2, \ldots$ Convergence results for inexact solutions of approximate problems to a solution of the limit non-stationary problem were established under rather mild conditions in [5].

Being based on the previous results, we suggest a two-level method for the basic VI (4), which collects consecutively the information about network users behavior. Each iteration of the method corresponds to some time slot t, whereas a lower level method solves the t-th approximate VI (5) within some tolerances. We take the gradient projection method (see e.g. [6], [3], [4]) with linesearch for this problem, it is called Algorithm P at Step 1 of the upper level method.

Upper Level Method.

Step 0. Let some number $\delta > 0$ and a sequence of positive numbers $\{\varepsilon_t\}$ (tolerances), and probability matrices $P^{(1,i)}$ for each $i \in \mathcal{N}$ be given. We set $t = 1$ and choose an initial point $y^{(1)} \in X^{(1)}$.

Step 1. Using the point $y^{(t)}$ we solve VI (5) by Algorithm P described below within the tolerance ε_t and obtain an approximate solution denoted by $x^{(t,\varepsilon_1)}$.

Step 2. We make the transition to the next state of the network. We find new positions of users and calculate the matrices $P^{(t+1),i}$ for all $i \in \mathcal{N}$.

Step 3. If $\|P^{(t,i)} - P^{(t+1),i}\| < \delta$ for all $i \in \mathcal{N}$, then we obtain the desired accuracy of calculations and stop the iterative process. Otherwise we set $y^{(t+1)} = x^{(t,\varepsilon_1)}$, $t = t + 1$ and go to Step 1.

We performed some numerical experiments on test problems with this method, varying parameters related to dimensionality and accuracy. They confirmed the applicability of the proposed approach to wireless networks with dynamic structure.

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