Worst-Case Weighted Sum-Rate Maximization for MISO Downlink Systems via Branch and Bound

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***Abstract*—We consider the worst-case weighted sum-rate maximization (WSRMax) problem under imperfect channel state information in multicell downlink multi-input single-output systems. The problem is known to be NP-hard. We propose a solution method, based on semi-definite relaxation and branch and bound technique, which solves globally the noncovex robust WSRMax problem with an optimality certificate. The proposed method can be easily extended to maximize any worst-case system performance metric that is Lipschitz continuous and increasing function of signal-to-interference-plus-noise ratio.**

I. Introduction

We consider the problem of weighted sum-rate maximization (WSRMax) for multicell downlink systems with linear precoding.The base stations (BSs) are assumed to have multiple antennas while all the receivers are equipped with single antenna. Further, imperfect channel state information (CSI) knowledge in all relevant channels are considered.

The WSRMax problem under the perfect CSI has been addressed in literatures, e.g., [1], [2]. However, in practice the BSs can never have perfect CSI, due to the imperfect channel estimation and limited feedback bandwidth. Such CSI errors at BSs can be modeled either by assuming that the channel errors are random and follow a certain statistical distribution [3], or by assuming that the CSI errors lie in a bounded uncertainty region [4]. In this paper, we consider bounded uncertainty region to model the imperfect CSI errors, and then design the beamfomers to optimize the worst-case WSRMax problem. Note that the WSRMax problem is an NP-hard problem, even for the perfect CSI case.

The main contribution of the paper is to propose a solution method, based on branch and bound (BB) technique and semi-definite programming (SDP), which solves globally the nonconvex worst-case WSRMax problem with channel uncertainties within a pre-defined accuracy ϵ. It is worth noting that this paper extends our recent work [5] to multi-input single-output (MISO) systems.

The proposed BB algorithm computes a sequence of asymptotically tight upper and lower bounds for the maximum worst-case weighted sum-rate, and it terminates when the difference between the upper and lower bound is smaller than ϵ. The efficiency of BB algorithm depends on computationally efficient lower and upper bound functions [6]. In [2] the perfect CSI is considered, thus the feasibility of the bounding functions can be easily checked by a second order cone program. In this paper, we consider imperfect CSI in all relevant channels, which makes the problem much difficult than that in [2].

II. System Model

A multicell MISO downlink system, with N BSs each equipped with T transmit antennas is considered. The set of all BSs is denoted by $\mathcal{N}$. We denote the set of all data streams in the system by $\mathcal{L}$. The transmitter node (i.e., the BS) of lth data stream is denoted by $tran(l)$ and the receiver node of lth data stream is denoted by $rec(l)$. We have $\mathcal{L} = \cup_{n\in\mathcal{N}}\mathcal{L}(n)$, where $\mathcal{L}(n)$ denotes the set of data streams transmitted by nth BS (see Figure 1).

The received SINR at $rec(l)$ is given by

$$\Gamma_l = \frac{|\mathbf{h}_{ll}^{\mathrm{H}}\mathbf{w}_l|^2}{\sigma_l^2 + \sum\limits_{\substack{j\in\mathcal{L}(tran(l))\\ j\neq l}}|\mathbf{h}_{ll}^{\mathrm{H}}\mathbf{w}_j|^2 + \sum\limits_{n\in\mathcal{N}\setminus\{tran(l)\}}\sum_{j\in\mathcal{L}(n)}|\mathbf{h}_{jl}^{\mathrm{H}}\mathbf{w}_j|^2}, \quad (1)$$

where $\mathbf{w}_l \in \mathbb{C}^T$ represent the transmit beamformer associated to lth data stream, $\mathbf{h}_{jl}^{\mathrm{H}} \in \mathbb{C}^{1\times T}$ denotes the channel vector from $tran(j)$ to $rec(l)$, and $n_l \sim \mathcal{CN}(0, \sigma_l^2)$.

A. Channel Uncertainty Model

We assume that the channels are uncertain at the network controller, but they belong to a known compact sets of possible values. We model the channel vector $\mathbf{h}_{jl}$ as

$$\mathbf{h}_{jl} = \hat{\mathbf{h}}_{jl} + \mathbf{e}_{jl}, \quad j,l \in \mathcal{L}, \quad (2)$$

where $\hat{\mathbf{h}}_{jl} \in \mathbb{C}^T$ denotes the estimated value of channel at the network controller and $\mathbf{e}_{jl} \in \mathbb{C}^T$ represents the corresponding channel estimation error. It is assumed that $\mathbf{e}_{jl}$ can take any value inside a T-dimensional complex ellipsoid, which is defined as $\mathcal{E}_{jl} = \left\{\mathbf{e}_{jl} : \mathbf{e}_{jl}^{\mathrm{H}}\mathbf{Q}_{jl}\mathbf{e}_{jl} \leq 1\right\}$, where $\mathbf{Q}_{jl}$ is a complex Hermitian positive definite matrix, assumed to be known, which specifies the size and shape of the ellipsoid. Now, with the channel uncertainty model (2), the received SINR (1) can be expressed as in (3) (in the top of next page).

B. Problem Formulation

Let β_l be an arbitrary nonnegative weight associated with data stream l. We consider the case where all receivers are using *single-user detection*. Assuming that the power allocation is subject to a maximum power constraint $\sum_{l\in\mathcal{L}(n)}\|\mathbf{w}_l\|_2^2 \leq p_n^{\max}$ for each BS $n \in \mathcal{N}$, the problem of worst-case WSRMax can be expressed as

$$\begin{array}{ll}\text{maximize} & \min_{\mathbf{e}_{jl}\in\mathcal{E}_{jl}, j,l\in\mathcal{L}} \sum_{l\in\mathcal{L}}\beta_l\log(1+\Gamma_l)\\ \text{subject to} & \sum_{l\in\mathcal{L}(n)}\|\mathbf{w}_l\|_2^2 \leq p_n^{\max}, \quad n\in\mathcal{N},\end{array} \quad (4)$$

with variables $\mathbf{e}_{jl}$ and $\mathbf{w}_l$ for all $j,l \in \mathcal{L}$.

$$\Gamma_l = \frac{|(\hat{\mathbf{h}}_{ll}+\mathbf{e}_{ll})^{\mathrm{H}}\mathbf{w}_l|^2}{\sigma_l^2 + \sum\limits_{j\,\in\,\mathcal{L}(tran(l)),\,j\,\neq\,l} |(\hat{\mathbf{h}}_{ll}+\mathbf{e}_{ll})^{\mathrm{H}}\mathbf{w}_j|^2 + \sum\limits_{n\in\mathcal{N}\backslash\{tran(l)\}} \sum_{j\in\mathcal{L}(n)} |(\hat{\mathbf{h}}_{jl}+\mathbf{e}_{jl})^{\mathrm{H}}\mathbf{w}_j|^2}. \quad (3)$$

III. Method

We start by equivalently reformulating problem (4) as

$$\begin{array}{ll} \text{minimize} & -\sum_{l\in\mathcal{L}} \beta_l \log(1+\gamma_l) \\ \text{subject to} & \gamma_l \leq \inf_{\mathbf{e}_{jl}\in\mathcal{E}_{jl}, j\in\mathcal{L}} \Gamma_l, \quad l\in\mathcal{L} \\ & \sum_{l\in\mathcal{L}(n)} \|\mathbf{w}_l\|_2^2 \leq p_n^{\max}, \quad n\in\mathcal{N}, \end{array} \quad (5)$$

with variables γ_l and $\mathbf{w}_l$ for all $j,l\in\mathcal{L}$. Let us define the objective function of problem (5) as $f_0(\boldsymbol{\gamma}) = \sum_{l\in\mathcal{L}} -\beta_l \log(1+\gamma_l)$ and the feasible set $\mathcal{G}$ for variables $\boldsymbol{\gamma}$ as

$$\mathcal{G} = \left\{ \boldsymbol{\gamma} \;\middle|\; \begin{array}{ll} \gamma_l \leq \inf\limits_{\{\mathbf{e}_{jl}|\mathbf{e}_{jl}^{\mathrm{H}}\mathbf{Q}_{jl}\mathbf{e}_{jl}\leq 1\}_{j\in\mathcal{L}}} \Gamma_l, & l\in\mathcal{L} \\ \sum_{l\in\mathcal{L}(n)} \|\mathbf{w}_l\|_2^2 \leq p_n^{\max}, & n\in\mathcal{N} \end{array} \right\}. \quad (6)$$

Then we apply *Algorithm 1* proposed in [2] to minimize function $f_0(\boldsymbol{\gamma})$ over the L-dimensional rectangle $\mathcal{Q}_{\text{init}}$, where $\mathcal{G}\subseteq\mathcal{Q}_{\text{init}}$. However, checking the condition $\boldsymbol{\gamma}\in\mathcal{G}$, which is central to calculating upper and lower bound functions in BB algorithm [2], is much more difficult in the case of uncertain CSI.

Checking the condition $\boldsymbol{\gamma}\in\mathcal{G}$ is equivalent to solving the following feasibility problem

$$\begin{array}{ll} \text{find} & \{\mathbf{w}_l, \mathbf{e}_{jl}\}_{j,l\in\mathcal{L}} \\ \text{subject to} & \gamma_l \leq \inf_{\mathbf{e}_{jl}\in\mathcal{E}_{jl}, j\in\mathcal{L}} \Gamma_l, \quad l\in\mathcal{L} \\ & \sum_{l\in\mathcal{L}(n)} \|\mathbf{w}_l\|_2^2 \leq p_n^{\max}, \quad n\in\mathcal{N}, \end{array} \quad (7)$$

with variables $\mathbf{w}_l$ and $\mathbf{e}_{jl}$, $j,l\in\mathcal{L}$. To solve problem (7) an equivalent reformulation is obtained and then the S-procedure [7] is used to handle the non-convex quadratic constraints due to channel uncertainties (first inequality constraints of (7)). However, this equivalent reformulation leads non-convex rank constraints in SDP formulation. It turns out that we can employ a simple trick to handle the rank constraints. The procedure is based on relaxing the rank constraints and introducing the objective function, minimization of transmit sum power of the network, in (7). Then the beamformer vectors can be found directly by eigen-decomposition of the optimal rank-one matrices, i.e., $\mathbf{W}_l^\star = \mathbf{w}_l^\star \mathbf{w}_l^{\star H}$.

IV. Simulation results

In the simulation multicell wireless network as shown in Figure 1 is considered. The channel matrix between BSs and users is modeled as $\mathbf{h}_{jl} = (d_{jl}/d_0)^{-\eta/2}\,\mathbf{c}_{jl}$, where the first term $(d_{jl}/d_0)^{-\eta/2}$ denotes the large scale fading, and the second term $\mathbf{c}_{jl}$ denotes the small scale fading. We define the signal-to-noise ratio (SNR) operating point at a distance r as $\text{SNR}(r) = (r/d_0)^{-\eta}\, p_0^{\max}/\sigma^2$.

Figure 2 shows the evolution of upper and lower bounds for the optimal value of problem (5) for β_l = 1 for all $l\in\mathcal{L}$, $p_0^{\max}/\sigma^2 = 40$dB, and $\text{SNR}(R_{\text{BS}}) = 10$dB. We assume that $\mathbf{Q}_{jl} = (1/\xi^2)\mathbf{I}$ and $\xi = 0.01$ for all $j,l\in\mathcal{L}$. Results show that the proposed algorithm converges with the number of iterations. Figure 3 compares the performance of the proposed robust algorithm and non-robust algorithm [2] for WSRMax in the presence of channel errors.

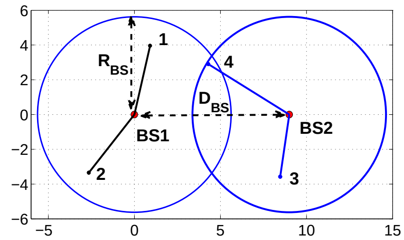

Fig. 1. MISO downlink wireless network with $N = 2$ BSs and $T = 4$ transmit antennas per BS. $\mathcal{L} = \{1,2,3,4\}$. $\mathcal{N} = \{1,2\}$, $\mathcal{L}(1) = \{1,2\}$, $\mathcal{L}(2) = \{3,4\}$.

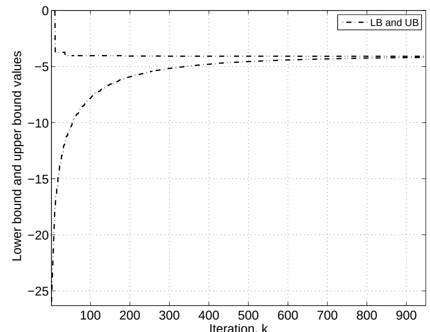

Fig. 2. Convergence of lower and upper bound values.

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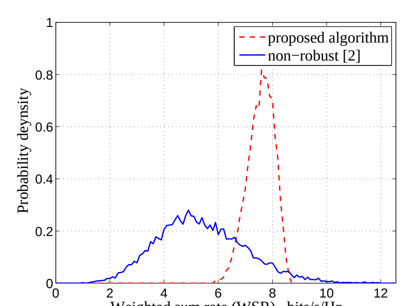

Fig. 3. PDF of achieved WSR for proposed robust and non-robust algorithms.