The Maximum Stable Throughput Region of the Two-User Interference Channel

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I. Introduction

In this paper, we consider the two-user interference channel, which models communication scenarios in which multiple one-to-one transmissions over a common frequency band are taking place creating interference one each other. The capacity region of the general Gaussian interference channel is a long standing problem and is only known for special cases, such as Gaussian channels with weak ("noisy") or strong interference [2]–[4]. Furthermore, information-theoretic results advocate for different ways of handling the interference, including orthogonal access, treating interference as noise (IAN), successive interference cancellation (SIC), joint decoding and interference alignment [5]. Here, we investigate the stability region of the two-user interference channel, which, to the best of our knowledge, has not been reported to the literature. In [6], the effect of multipacket reception on stability and delay of slotted ALOHA-based random access systems is considered. In [7], the authors studied a cognitive interference channel, as well as the case of a primary user and a cognitive user with and without relaying capabilities. The maximum stable throughput of the cognitive user for a fixed throughput selected by the primary user is derived.

In this work, we investigate the two-user interference channel, where each user has bursty arrivals and transmits a packet whenever its queue is not empty, and we obtain the exact stability region for the general case. The characterization of the stability region is a challenging problem due to the fact that the user queues are coupled, i.e. the service process of a queue depends on the status of the other queues. To overcome this difficulty, the stochastic dominance technique is used here [8]. We also consider the cases where each receiver treats interference as noise or employ successive interference cancelation. Finally, we present conditions for the shape of the stability region (concave or convex).

II. System Model

We consider a two-user interference channel, as depicted in Fig. 1, in which each source $S_i, i = 1, 2$ intends to communicate with its respective destination $D_i, i = 1, 2$. The packet arrival processes at S_1 and S_2 are assumed to be independent and stationary with mean rates λ_1 and λ_2, respectively. Transmitter S_i has an infinite capacity queue to store incoming packets and Q_i denotes the size in number packets of the i-th queue. The transmission rates of S_1 and S_2 are fixed at R_1 and R_2, respectively.

Time is assumed to be slotted and each source transmits a packet in a timeslot if its queue is not empty; otherwise it remains silent. The transmission of one packet requires one timeslot and we assume that ACKs are instantaneous and error-free. A block fading channel model is considered here with Rayleigh fading, i.e. the fading coefficients h_{ij} remain constant during one timeslot, but change independently from one timeslot to another based on a circularly symmetric complex Gaussian distribution with zero mean and unit variance. The noise is assumed to be additive white Gaussian with zero mean and unit variance. With p_i we denote the transmission power of source S_i, and r_{ij} is the distance between transmitter S_i and receiver D_j with a being the path loss exponent.

This work [1] was presented in IEEE ITW 2013, Sevilla, Spain.

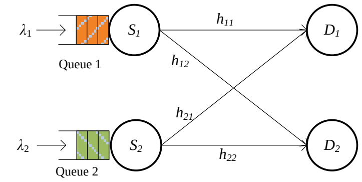

Fig. 1: Two-user interference channel with bursty arrivals.

Let $\mathcal{D}_i^{\mathcal{T}}$ denote the event that destination i is able to decode the packet transmitted from the i-th source given a set of active transmitters denoted by $\mathcal{T}$ i.e. $\mathcal{D}_1^{\{1,2\}}$ denotes the event that the first destination can decode the information from the first source when both transmitters are active ($\mathcal{T} = \{1, 2\}$). When only S_i is active the event $\mathcal{D}_i^{\{i\}}$ is defined as

$$\mathcal{D}_i^{\{i\}} \triangleq \left\{R_i \leq \log_2\left(1 + |h_{ii}|^2 r_{ii}^{-a} p_i\right)\right\}. \quad (1)$$

For convenience we define $\mathrm{SNR}_i \triangleq |h_{ii}|^2 r_{ii}^{-a} p_i$ and $\gamma_i \triangleq 2^{R_i} - 1$. The probability that the link ii is not in outage when only S_i is active is given by [9]

$$\Pr\left(\mathcal{D}_i^{\{i\}}\right) = \Pr\left\{\mathrm{SNR}_i \geq \gamma_i\right\} = \exp\left(-\frac{\gamma_i r_{ii}^a}{p_i}\right). \quad (2)$$

The events $\mathcal{D}_i^{\{i,j\}}$ (both sources are active) are defined based on the specific interference treatment on each receiver.

We adopt the definition of queue stability used in [10].

Definition 1. *Denote by Q_i^t the length of queue i at the beginning of time slot t. The queue is said to be* stable *if*

$$\lim_{t\to\infty} Pr[Q_i^t < x] = F(x) \textit{ and } \lim_{x\to\infty} F(x) = 1. \quad (3)$$

If $\lim_{x\to\infty} \lim_{t\to\infty} \inf Pr[Q_i^t < x] = 1$, the queue is substable. *If a queue is stable, then it is also substable. If a queue is not substable, then we say it is unstable.*

Loynes' theorem [11] states that if the arrival and service processes of a queue are strictly jointly stationary and the average arrival rate is less than the average service rate, then the queue is stable. The stability region of the system is defined as the set of arrival rate vectors (λ_1, λ_2) for which the queues in the system are stable.

III. Main Results

The stability region in a parametric form without considering any specific technique for treating the interference at the receivers for the two-user interference channel is given by $\mathcal{R} = \mathcal{R}_1 \bigcup \mathcal{R}_2$ where $\mathcal{R}_1$ and $\mathcal{R}_2$ are given by (4) and (5) respectively.

$$\mathcal{R}_1 = \left\{ (\lambda_1,\lambda_2) : \frac{\lambda_1}{\Pr\left(\mathcal{D}_1^{\{1\}}\right)} + \frac{\left[\Pr\left(\mathcal{D}_1^{\{1\}}\right) - \Pr\left(\mathcal{D}_1^{\{1,2\}}\right)\right]\lambda_2}{\Pr\left(\mathcal{D}_1^{\{1\}}\right)\Pr\left(\mathcal{D}_2^{\{1,2\}}\right)} < 1, \lambda_2 < \Pr\left(\mathcal{D}_2^{\{1,2\}}\right) \right\} \tag{4}$$

$$\mathcal{R}_2 = \left\{ (\lambda_1,\lambda_2) : \frac{\lambda_2}{\Pr\left(\mathcal{D}_2^{\{2\}}\right)} + \frac{\left[\Pr\left(\mathcal{D}_2^{\{2\}}\right) - \Pr\left(\mathcal{D}_2^{\{1,2\}}\right)\right]\lambda_1}{\Pr\left(\mathcal{D}_2^{\{2\}}\right)\Pr\left(\mathcal{D}_1^{\{1,2\}}\right)} < 1, \lambda_1 < \Pr\left(\mathcal{D}_1^{\{1,2\}}\right) \right\} \tag{5}$$

$$\mathcal{R}_1^{\mathrm{IAN}} = \left\{ (\lambda_1,\lambda_2) : \frac{\lambda_1}{\exp\left(-\frac{\gamma_1 r_{11}^a}{p_1}\right)} + \frac{\gamma_1 \frac{p_2}{p_1}\left(\frac{r_{11}}{r_{21}}\right)^a + \gamma_1\gamma_2\left(\frac{r_{11}r_{22}}{r_{12}r_{21}}\right)^a}{\exp\left(-\frac{\gamma_2 r_{22}^a}{p_2}\right)}\lambda_2 < 1, \lambda_2 < \frac{\exp\left(-\frac{\gamma_2 r_{22}^a}{p_2}\right)}{\left[1+\gamma_2\frac{p_1}{p_2}\left(\frac{r_{22}}{r_{12}}\right)^a\right]} \right\} \tag{6}$$

$$\mathcal{R}_2^{\mathrm{IAN}} = \left\{ (\lambda_1,\lambda_2) : \frac{\lambda_2}{\exp\left(-\frac{\gamma_2 r_{22}^a}{p_2}\right)} + \frac{\gamma_2 \frac{p_1}{p_2}\left(\frac{r_{22}}{r_{12}}\right)^a + \gamma_1\gamma_2\left(\frac{r_{22}r_{11}}{r_{12}r_{21}}\right)^a}{\exp\left(-\frac{\gamma_1 r_{11}^a}{p_1}\right)}\lambda_1 < 1, \lambda_1 < \frac{\exp\left(-\frac{\gamma_1 r_{11}^a}{p_1}\right)}{\left[1+\gamma_1\frac{p_2}{p_1}\left(\frac{r_{11}}{r_{21}}\right)^a\right]} \right\} \tag{7}$$

$$\mathcal{R}_1^{\mathrm{SIC}} = \left\{ (\lambda_1,\lambda_2) : \frac{\lambda_1}{\Pr\{\mathrm{SNR}_1 \geq \gamma_1\}} + \frac{1-\Pr\{\{\mathrm{SINR}_{21}\geq\gamma_2\} \mid \{\mathrm{SNR}_1\geq\gamma_1\}\}}{\Pr\{\{\mathrm{SINR}_{12}\geq\gamma_1\}\cap\{\mathrm{SNR}_2\geq\gamma_2\}\}}\lambda_2 < 1, \lambda_2 < \Pr\{\{\mathrm{SINR}_{12}\geq\gamma_1\}\cap\{\mathrm{SNR}_2\geq\gamma_2\}\} \right\} \tag{8}$$

$$\mathcal{R}_2^{\mathrm{SIC}} = \left\{ (\lambda_1,\lambda_2) : \frac{\lambda_2}{\Pr\{\mathrm{SNR}_2 \geq \gamma_2\}} + \frac{1-\Pr\{\{\mathrm{SINR}_{12}\geq\gamma_1\} \mid \{\mathrm{SNR}_2\geq\gamma_2\}\}}{\Pr\{\{\mathrm{SINR}_{21}\geq\gamma_2\}\cap\{\mathrm{SNR}_1\geq\gamma_1\}\}}\lambda_1 < 1, \lambda_1 < \Pr\{\{\mathrm{SINR}_{21}\geq\gamma_2\}\cap\{\mathrm{SNR}_1\geq\gamma_1\}\} \right\} \tag{9}$$

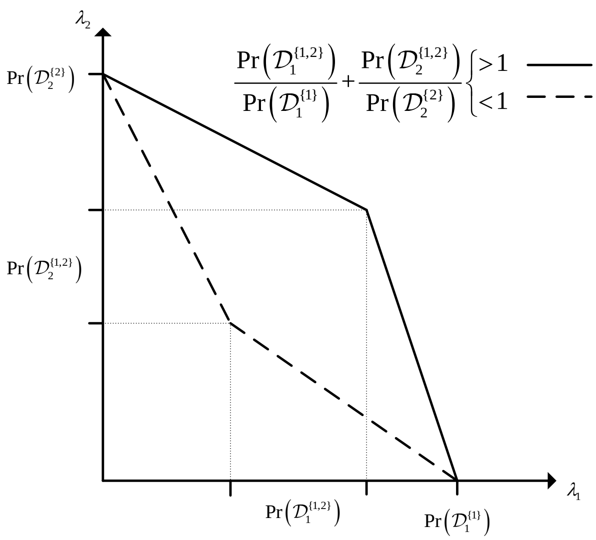

Fig. 2: The stability region for the general case.

It is easy to see that if

$$\frac{\Pr\left(\mathcal{D}_1^{\{1,2\}}\right)}{\Pr\left(\mathcal{D}_1^{\{1\}}\right)} + \frac{\Pr\left(\mathcal{D}_2^{\{1,2\}}\right)}{\Pr\left(\mathcal{D}_2^{\{2\}}\right)} \gtrless 1, \tag{10}$$

then the stability region is concave/convex and is depicted in Fig. 2.

The stability region when both destinations decode their individual messages by treating the interference from unintended sources as noise is $\mathcal{R}^{\mathrm{IAN}} = \mathcal{R}_1^{\mathrm{IAN}} \cup \mathcal{R}_2^{\mathrm{IAN}}$. $\mathcal{R}_1^{\mathrm{IAN}}$ and $\mathcal{R}_2^{\mathrm{IAN}}$ are given by (6) and (7) respectively. $\mathcal{R}^{\mathrm{IAN}}$ is convex/concave when:

$$\gamma_1\gamma_2 \lesseqgtr \left(\frac{r_{12}r_{21}}{r_{22}r_{11}}\right)^a. \tag{11}$$

When both receivers employ successive interference cancelation when both transmitters are active, the stability region is $\mathcal{R}^{\mathrm{SIC}} = \mathcal{R}_1^{\mathrm{SIC}} \cup \mathcal{R}_2^{\mathrm{SIC}}$, where $\mathcal{R}_1^{\mathrm{SIC}}$ and $\mathcal{R}_1^{\mathrm{SIC}}$ are given by (8) and (9) respectively. The $\mathcal{R}^{\mathrm{SIC}}$ is concave/convex if

$$\Pr\{\{\mathrm{SINR}_{21}\geq\gamma_2\} \mid \{\mathrm{SNR}_1\geq\gamma_1\}\} + \Pr\{\{\mathrm{SINR}_{12}\geq\gamma_1\} \mid \{\mathrm{SNR}_2\geq\gamma_2\}\} \gtrless 1.$$

The complete proofs for the previous results are given in [1]. Also in [1] are presented the conditions for which a certain interference management technique leads to broader stability region compared to the others.

IV. Conclusions

We derived the stability region of the two-user interference channel for the general case and for different interference management strategies, namely treating interference as noise and successive interference cancelation at the receivers. Furthermore, we provided conditions for the convexity/concavity of the stability regions.

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